1. Consider a LTI discrete system which is described by the following equation.

$$H(e^{j\omega}) = \begin{cases} 0, & |\omega| < 0.5\pi \\ 1, & 0.5\pi \le |\omega| < \pi \end{cases}$$

The input to this system is

$$x[n] = \cos(0.6\pi n) - \sin(0.4\pi n) + 2$$

Determine the output y[n].

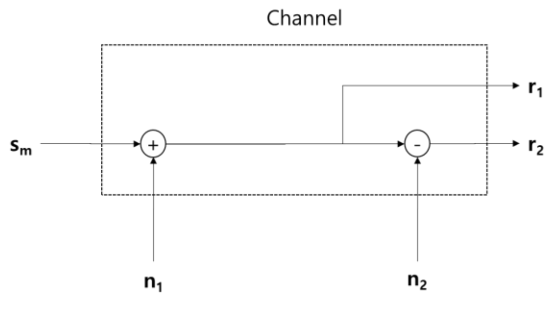
2. Show that

$$\sum_{n=-\infty}^{\infty} |h[n]|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left|H(e^{j\omega})\right|^2 d\omega$$

3. Suppose a discrete-time source is modelled by a zero-mean Gaussian process $\{X_n\}$. Compare the magnitude of variances between $(X_n - X_{n-1})$ and $X_n$.

4. For a given $x_l(t)$, define $x(t) = Re\{x_l(t)e^{j2\pi f_0 t}\}$ and $\hat{x}(t) = Im\{x_l(t)e^{j2\pi f_0 t}\}$. Find the Fourier transforms of $x(t)$ and $\hat{x}(t)$ in terms of $X_l(f) = F\{x_l(t)\}$.

5. In the communication system below, Rx receives two signals $\mathbf{r_1}$ and $\mathbf{r_2}$.




If the two additive noises $\mathbf{n_1}$ and $\mathbf{n_2}$ are independent or dependent, show whether $\mathbf{r_1}$ is a sufficient statistic for optimal (i.e. MAP) detection or not.

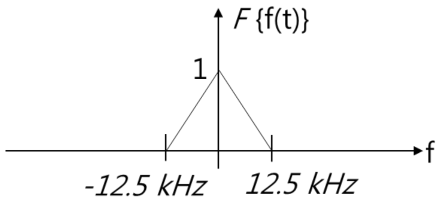
6. A random process is defined by

$$x(t) = aAcos(2\pi f_0 t) - bBsin(2\pi f_0 t)$$

Where A and B are two zero-mean independent Gaussian random variables with variances

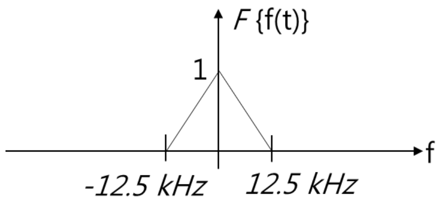
$\sigma_A^2$ and $\sigma_B^2$, respectively, a and b are non-zero constants. Find the condition that x(t) is wide-sense stationary.

7. Consider an analog signal, f(t), that is band-limited to $\pm$ 12.5 kHz. Its spectrum is as follows.



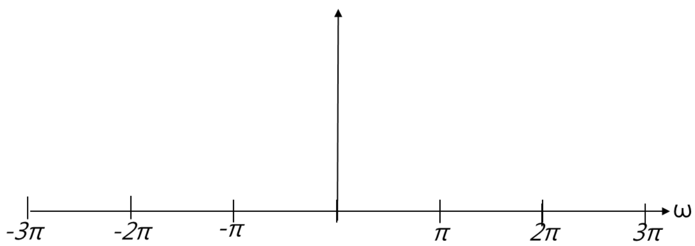

Determine the minimum sampling frequency in the unit of Hz and the maximum sampling period to avoid the aliasing effect

8. Consider an analog signal, f(t), that is band-limited to $\pm$ 12.5 kHz. Its spectrum is as follows.

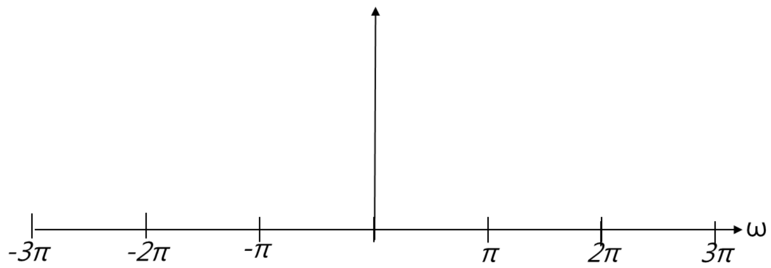



The analog signal, f(t), is sampled to obtain the discrete-time signal, $x_1[n]$ (with a sampling time 20 μs), and $x_2[n]$ (with a sampling time 40 μs). Plot the frequency spectra of $x_1[n]$ and $x_2[n]$ from -3π to 3π. (Indicate the magnitude and cut-off frequency.)

$X_1(e^{j\omega}) = F\{x_1[n]\}$




$X_2(e^{j\omega}) = F\{x_2[n]\}$




9. The communication system at 10.0 GHz, with a transmit antenna gain of 34 dB, an IF bandwidth of 10 MHz, and a distance from Tx to Rx 100 km. The 18-inch receiving dish antenna has a gain of 33.5 dB. Find the path loss in dB.

10. The communication system at 10.0 GHz, with a transmit antenna gain of 34 dB, an IF bandwidth of 10 MHz, and a distance from Tx to Rx 100 km. The 18-inch receiving dish antenna has a gain of 33.5 dB and sees an overall receiver system noise temperature is $T_{sys}$ = 700 K. When the required minimum SNR in the receiver is 15 dB, and the link margin of the system is 10 dB, determine the transmitted signal power in the unit of mW. (Receiver noise power = $kT_{sys}B$)