계산이론 (Theory of Computation)

1. For the alphabet $\sum=\{0,1\}$, find a DFA that accepts all strings not ending 01. For example, the DFA should accept 11, and reject 001.

2. For the alphabet $\sum=\{0,1\}$, find a DFA for the language accepted by the odd parity checker. For example, the DFA should accept 0001, and reject 0011.

3. Find a regular expression which is equivalent to the DFA found in the problem 2.

4. Show that regular languages are closed under intersection.

5. Determine whether or not the following grammar is ambiguous. Explain the reason. (20%)

   $S \rightarrow abS \mid aSb \mid \lambda$

6. Convert the grammar into Chomsky normal form. (20%)

   $S \rightarrow SS \mid bSa \mid aSb \mid \lambda$

7. Convert the grammar into Chomsky normal form. (20%)

   $S \rightarrow aSbS \mid bSaS \mid \lambda$

8. Find a context free grammar for the language:

   $L = \{ a^n b^m \mid n \neq m \}$.

9. Find a context free grammar for the language:

   $L = \{ a^n b^{n+m} a^m \mid n, m > 0 \}$.

10. Find a context free grammar for the language:

    $L = \{ a^n b^m \mid n > 2m \}$.

11. Determine whether or not the language $L=\{ ww : w \in \{a,b\}^* \}$ is regular. Then, explain the reason.

12. Determine whether or not the language $L=\{ 0^n 1^n \mid n > 0 \}$ is regular. Then, explain the reason.

13. The symmetric difference of two sets S1 and S2 is defined as

S1 ⊖ S2 = { x: x ∈ S1 or x ∈ S2, but x is not in both S1 and S2}.

Determine whether or not context free languages are closed under the symmetric difference.

14. Show that context-free languages are not closed under intersection.

15. Determine whether or not the language L={ ww : w ∈ {a,b}* } is regular. Then, explain the reason.

16. Explain the Chomsky hierarchy.

17. Explain the Church-Turing's Thesis.

18. What is your definition of "computation" ?

19. What are the differences between NFA and DFA?

20. Given two DFAs, M1 and M2, how do you show that
M1 = M2 or M1 =/= M2 ?

21. Pumping Lemma를 사용하여 $L = \{a^n b^n : n \geq 0\}$ 이 regular가 아님을 증명하라.

22. $\Sigma = \{a, b\}$에 대하여, a가 최대 세 개 포함된 모든 string을 accept하는 DFA를 그려라.

23. 다음 문제를 풀어라

A. Context-free 언어, context-sensitive 언어 등의 용어에서, “context”란 무엇을 구체적으로 지칭하는가?

B. 문법 G=(V,T,S,P) 에서, A ∈ V 이며 x ∈ $(V \cup T)^*$이다 (V는 nonterminal 집합, T는 terminal 집합). 이 때, Context-free인 언어의 production rule의 예를 하나 들고,

위의 (A)번 답에 의거 context-free의 의미를 설명하라.

24. Recursively enumerable, regular, context-free, context-sensitive 언어 사이의 Chomsky Hierarchy를 동심원으로 표현하라.

25. Use the CYK algorithm to determine if the string *aab* is in the language generated by the following grammar:

    S → AB

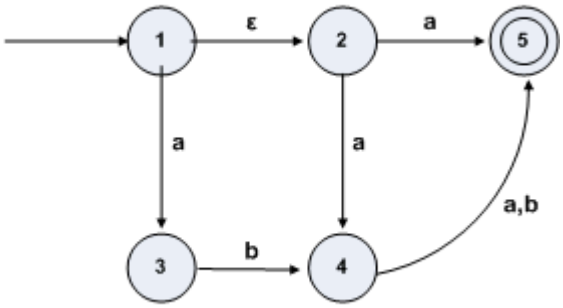
    A → BB | a

    B → AB | b

26. Construct an npda for accepting the language $L = \{ww^R \mid w \in \{a,b\}^+\}$, where $Q = \{q_1, q_2, q_3\}$, $\Sigma = \{a,b\}$, $\Gamma = \{a,b,z\}$, $F = \{q_2\}$. Hint: define the transition function δ.

27. Construct an dpda $M = (\{q_0, q_1, q_2\}, \{a,b\}, \{0,1\}, \delta, q_0, 0, \{q_0\})$ that accepts $L = \{a^n b^n \mid n \geq 0\}$. Hint: again, define δ.

28. Construct an equivalent DFA for the following NFA



Q = states = {1,2,3,4,5}
Start state: { 1 }
Accepting state(s): { 5 }

29. Create an NFA that accepts the same language generated by $((0+01)^*11^*(0+11)^*)^*$. Convert that NFA to a DFA.

30. For each of the context-free languages below, create a grammar that generates the language that is in Chomsky Normal Form (if the language contains the empty string then your grammar may omit it).

    C. $\{w \in \{a,b\}^* \mid$ the length of w is even and the first half is all a's$\}$

    D. $\{w \in \{0,1\}^* \mid$ the number of times 01 appears as a substring is equal to the number of times 10 appears as a substring$\}$

31. Turing Decidable, Turing Undecidable 이란 무엇인지 설명하라.

32. Church의 가설 (Church's Thesis) 가 무엇인지 설명하고, Turing Undecidable 문제를 사람의 computation 관점에서 설명하라.

33. DFA M 이 주어졌을 때, M의 computation 에 대해 설명하라.

34. Halting Problem 에 대해 설명하라.

35. 심볼의 집합 Σ가 주어졌을 때, 언어(Language)란 무엇인지 설명하라.

36. 심볼의 집합 Σ과 언어 L 이 주어졌을 때, 임의의 문자열 $w \in \Sigma^*$ 가 L의 원소인지 아닌지 항상 컴퓨터를 이용하여 대답을 할 수 있는 지 설명하라.

37. Theory of Computation 이란 무엇인가?

38. "인간은 어떠한 문제에 대해서도 대답할 수 있다" 라는 명제에 대해 computation 관점에서 설명하라. (물론 무한의 메모리와 속도가 빛의 속도인 컴퓨터를 이용해도 된다는 가정을 해도 된다)

39. Algorithm 이란 무엇인지 Turing Machine 관점에서 설명하라. (알고리즘의 속성 5가지를 튜링 기계의 관점으로 설명해도 됨)

40. Turing Machine으로 해결할 수 없는 Decision Problem 을 해결할 수 있는 새로운 기계가 있다면, 어떠한 변화가 생길 지 설명하라.

41. Halting Problem 의 정의를 설명하고 이 문제가 undecidable 임을 증명하라.

42. $L = \{a^n b^n \mid n \geq 0\}$ 이 decidable 임을 증명하라.

43. Draw a Venn-diagram to illustrate the relationships between the following classes of languages:

    A. RL: the set of regular languages

B. CFG: the set of context-free languages

C. UCFL: the set of context-free languages that have unambiguous grammars

D. PDA: the set of languages accepted by some pushdown automata

E. DPDA: the set of languages accepted by some deterministic pushdown automata

44. True/False questions:

A. Every context-free language is regular.

B. Every regular language is context-free.

C. If a language L satisfies the pumping lemma, then L is regular.

D. There exists a language that is not regular but satisfies the pumping lemma.

E. The syntax of programming languages like C and Java can be expressed by a regular expression.

F. The concatenation of two regular languages is regular.

G. There exists a general algorithm to remove ambiguity from a context-free grammar.

H. All computer programs written in modern programming languages can be implemented by some Turing machine.

I. The number of undecidable problems is countably infinite.

J. Every Turing machine can be represented by an expression in lambda calculus.